

# MATHEMATICAL EDUCATION ON MERSEYSIDE

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## **Senior Challenge '19**

### ***Year 10 or below***

Illustrations by Theo Chaddock and Will Ashworth

#### Rules

- 1) Senior Challenge '19 should be attempted at home during February half term.
- 2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
- 3) Entries without any working out at all or written on this sheet **will not be marked**.
- 4) It is possible to win a prize or certificate even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 5) Please make sure that you staple your pages together and you must write **your name and school neatly on every page**.

Either you or your maths teacher needs to return your entry by 8<sup>th</sup> March to this address:

Senior Challenge '19 Entries,  
Chris Marchant,  
Department of Mathematical Sciences,  
University of Liverpool,  
Peach Street,  
Liverpool,  
L69 7ZL

A Prize-Giving Evening will be held at the University of Liverpool on 8<sup>th</sup> May.  
We hope that you enjoy the questions.

### 1. Flashcard Flurry

Sue has produced flashcards in preparation for her revision of the noble gases for her Chemistry exam. In the correct order, they are He, Ne, Ar, Kr, Xe and Rn.

Being clumsy, she drops her flashcards. She picks them up & they now read Kr, Rn, Ne, Ar, He & Xe.

By swapping 2 adjacent cards at a time, what is the lowest number of swaps it will take for her to put them back as they should be?

Illustrate your solution swap-by-swap.



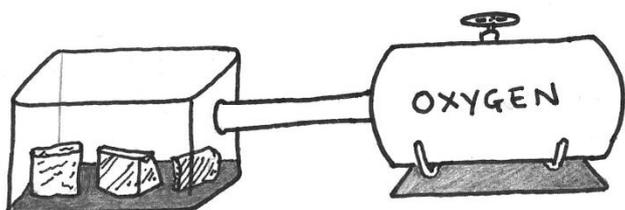
### 2. Bottles Banked

Jo bought bottles of dihydrogen monoxide on a month-by-month basis. One month, she received 45p change from a £5 bank note. The following month, there had been a drought, resulting in a price rise. This time, she received 50p change. How many bottles did she buy each time and what was the price? (At the time, dihydrogen monoxide cost between 60 and 80 pence per bottle.)



### 3. Fading Fermium

Half-life is the time taken for half of a quantity of a substance to decay. Fermium (atomic number 100) has 19 isotopes, most of which have half-lives of less than a second. On 31st August, after being put through a centrifuge to separate off the Fermium-257 (100-day half-life), Michael is left with 100kg of Fermium comprised of 40% Fermium-252 (half-life of 1 day) and 60% Fermium-253 (half-life of 3 days). To the nearest gram, what mass of Fermium will be left on 15th September?



### 4. Oxygen Consumption

Martin discovers that a block of sodium reacts with oxygen in a tank and exhausts the supply of oxygen in 3 minutes. Potassium achieves this in 4 minutes and magnesium in 6 minutes. How long would it take to use up all the oxygen in the tank if 1 block of each of the 3 metals were placed inside simultaneously? Assume that the reactions do not affect one another.



### 5. Charlie's Chemical Calamity

Charlie is carrying a tray with 3 bottles of element A, 3 bottles of element B, and 3 bottles of element C. Elements A and B react together to cause a modest explosion; however, the presence of C will always totally neutralise the reaction and prevent such an explosion. Unfortunately, Charlie trips! 3 of the bottles smash randomly on the floor with their contents mixing together instantaneously. As a fraction in its lowest terms, what is the probability that an explosion occurs?

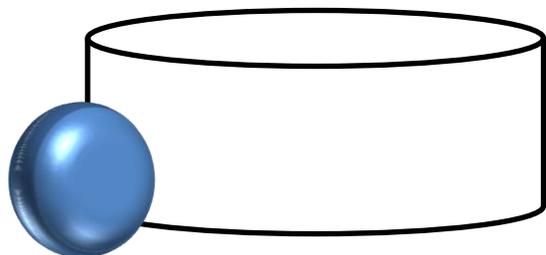
### 6. Lamentable Labelling

Helen asks one of her PhD students to label 4 bottles for her. Each bottle contains a different chemical. He forgets which is which, panics, and labels them at random. When Helen returns and uses the chemicals, she discovers that each bottle contains the wrong substance. What is the probability of this having happened?



### 7. Marble Pyramid

For its weight, titanium is a very strong metal. Robert places titanium marbles with a diameter of 2cm into a cylinder of diameter 6cm. How many marbles can he fit into a layer at the bottom of the cylinder? He then adds in a second layer, such that each marble in the second layer is touching 3 marbles in the first layer. How far apart are the centres of these 3 marbles? Finally, he adds a single marble as a third layer. How far above the base of the cylinder is the top of this final marble?

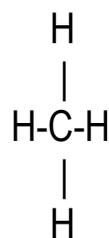


(Hint:  $\cos(30^\circ) = \frac{\sqrt{3}}{2}$ )

### 8. Breaking Bonds

Ann has bought herself a new mass-spectrometer. To check the machine works correctly, she runs it on  $\text{CH}_4$ , a chemical for which she knows the spectrum of masses of molecules and fragments of molecules it will contain. When she places a chemical in the machine, she knows that all, some, or none of the bonds (shown as solid lines in the diagram on the right) may break. How many different ways can this happen, considering all rotations and reflections separately? Assume multiple breaks are possible. What would the results be for  $\text{C}_2\text{H}_6$ ?

Methane ( $\text{CH}_4$ )



Ethane ( $\text{C}_2\text{H}_6$ )

