MATHEMATICAL EDUCATION ON MERSEYSIDE

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Senior Challenge '18
For Year 10 or below

Illustrations by Theo Chaddock and Will Ashworth

Rules
1) Senior Challenge '18 should be attempted at home during February half term.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school neatly on every page.**

Either you or your maths teacher needs to return your entry by 9th March to this address:
Senior Challenge '18 Entries,
Chris Marchant,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool,
L69 7ZL

A Prize-Giving Evening will be held at the University of Liverpool on 2nd May.
We hope that you enjoy the questions.

1. Who Packed all the Pies?

Kane's bakery makes four flavours of pie. The weight of each flavour pie is follows:

Apple pie – 700g
Banoffee pie – 725g
Custard pie – 600g
Rhubarb pie – 675g

Robert makes an order for 4 apple pies, 8 banoffee pies, 3 custard pies and 5 rhubarb pies to sell at a fundraiser.

Kane packs the order into boxes that can each hold 4kg. What is the fewest number of boxes he can pack the order into? Show your solution.
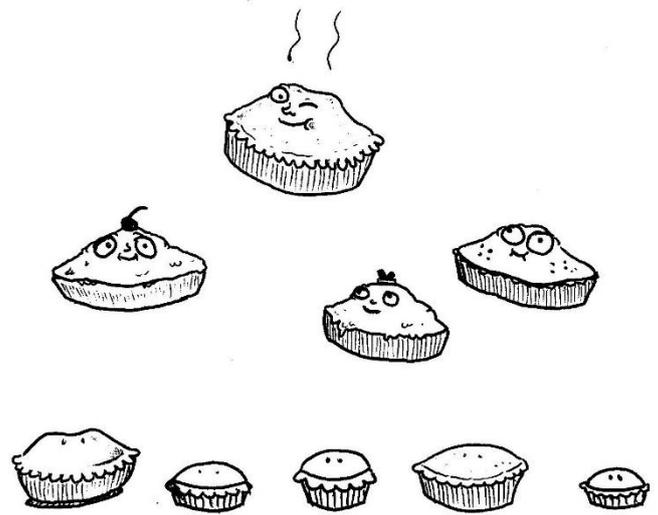
2. PB and J Array

Ken has made a selection of square jam sandwiches for a party, which he arranges in a single layer on the table in a perfect rectangle. Sarah has brought along the same number of square peanut butter sandwiches, which she puts out, also as a single layer, to form a border (one sandwich wide) around Ken's rectangle. Given that all of the sandwiches are the same size and they are all used, how many sandwiches could each person possibly have made?
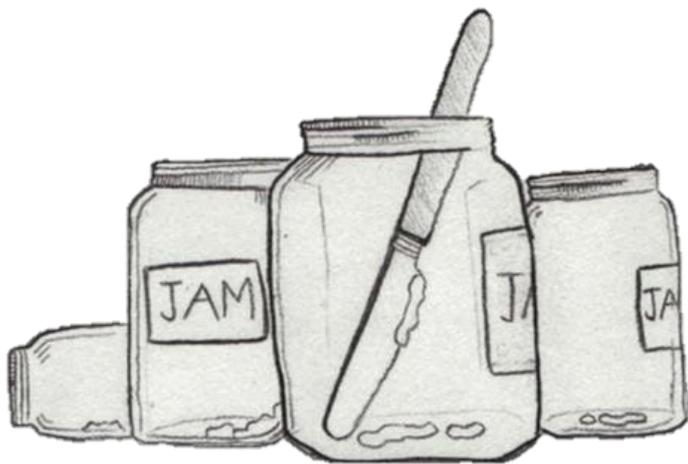
3. A Bonanza of Biscuits

Marianne has been collecting donations for her biscuit stall at the school summer fayre. There are some luxury gift tins of biscuits to be sold at £5 each, normal packets at £1 each, and mini-packs of 2 biscuits at 10p each. She tells Amy that she has received exactly 100 donations in total, with a collective value of £100, and that her stock of £1 packets is very low compared with the other items. Amy wants to work out how many of each item Marianne has. Show how she can do it.

4. Sun Dried Tomatoes

Michael picks 1kg of tomatoes from his greenhouse. He knows that fresh tomatoes are 95% water by weight.

He puts them on the window ledge to ripen, but forgets about them. After a week, he discovers that they have dried out and are now only 90% water by weight.

How much do the tomatoes now weigh?
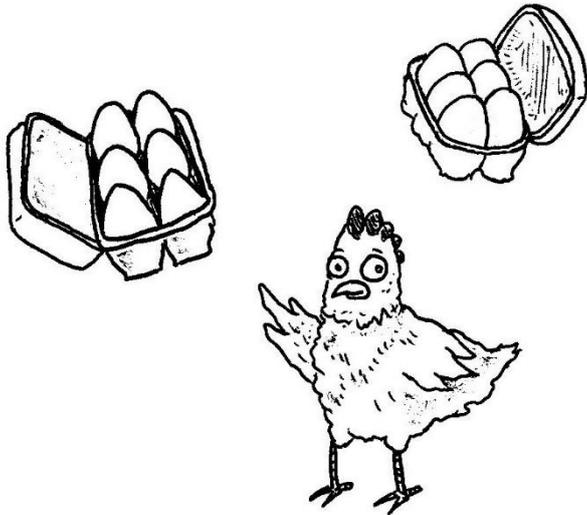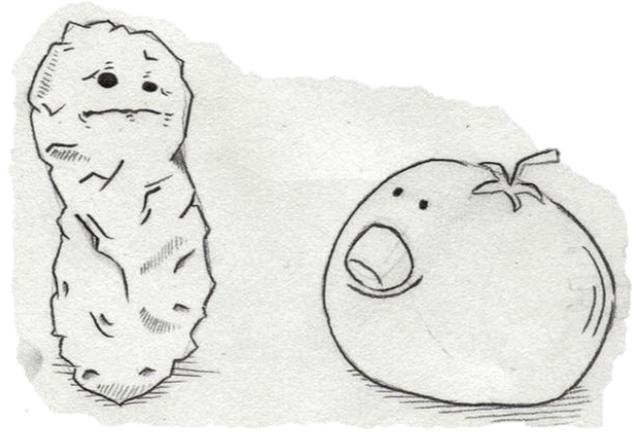
5. An Eggscellent Problem

In Ginoni's farm shop, you can buy her free-range hens' eggs in boxes of 4, 6, 9 or 20. This means, for example, that you can have 21 eggs by buying two boxes of 6 and a box of 9. List the quantities of eggs that are impossible to buy from her shop.

Ginoni notices that sales of the 4-egg boxes are poor, so discontinues that size. What is now the highest number of eggs that you can't buy? Prove that all larger quantities are possible.

6. Cartesian Cola Conundrum

Cartesian Cola is a very popular drinks brand among mathematicians. They sell two products.

A bottle of Cartesian Cola Classic contains 500ml water, 600g sugar, and 0.1g caramel.

A bottle of Cartesian Cola Quantum contains 550ml water, 200g sugar, and 0.2g caramel.

Even though demand for their products is virtually unlimited, every day they are limited to 100 litres of water, 100kg sugar, and 32g caramel. For each bottle sold, Cartesian Cola Classic makes 40p profit, and Cartesian Cola Quantum makes 19p profit.

How many bottles of each should Cartesian Cola produce each day to maximise their profit, and what is the profit per day?

7. Guess the Weight of the Cake

Jo has baked 40 cakes for her husband's 40th birthday, one for each of his years, weighing 1, 2, 3, … 40kg. She forgets which cake is which and has to check the weight of each cake using a large pair of balancing scales. Each cake is placed individually on the left-hand pan of the scales and balanced against one or more scale-weights, which can be put on either or both scale pans as necessary. She has four scale-weights available, each a different whole number of kg. Find their values in kg and make a list showing the particular combination of the scale-weights for identifying each cake.

8. No Such Thing as a Free Lunch?

Phoebe plans to eat out for lunch every day for two weeks, choosing from a row of 9 waterfront restaurants. She promises her brother Will that, if he picks one restaurant per day and manages to find her there, she will buy his meal for him. She intends to move venues daily but, to make Will's life a little easier, she will always move just one restaurant to the left or to the right, choosing her direction at random (unless at either end of the row, when her next move is forced). Will is not aware at which restaurants Phoebe has eaten. Either use a diagram to illustrate Will's strategy for finding Phoebe within the given two weeks and thus guaranteeing him a free lunch or else show that no such strategy exists.