

# MATHEMATICAL EDUCATION ON MERSEYSIDE

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# Senior Challenge '17 For Year 10 or below

Illustrations by Theo Chaddock and Will Ashworth

## Rules

- 1) Senior Challenge '17 should be attempted at home during February half term.
- 2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
- 3) Entries without any working out at all or written on this sheet **will not be marked**.
- 4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 5) You must write **your name and school in neat writing on every page**.

Either you or your maths teacher needs to return your entry by 10<sup>th</sup> March to this address:

Senior Challenge '17 Entries,  
Chris Marchant,  
Department of Mathematical Sciences,  
University of Liverpool,  
Peach Street,  
Liverpool,  
L69 7ZL

A Prize-Giving Evening will be held at the University of Liverpool on 3<sup>rd</sup> May.  
We hope that you enjoy the questions.

### 1. Fairly Hungry

Anna, Charlie and Michael are hungry at the fair. They buy a candy floss, a hot dog, and a gourmet burger, respectively.

They meet Sue, who asks the cost of each item.

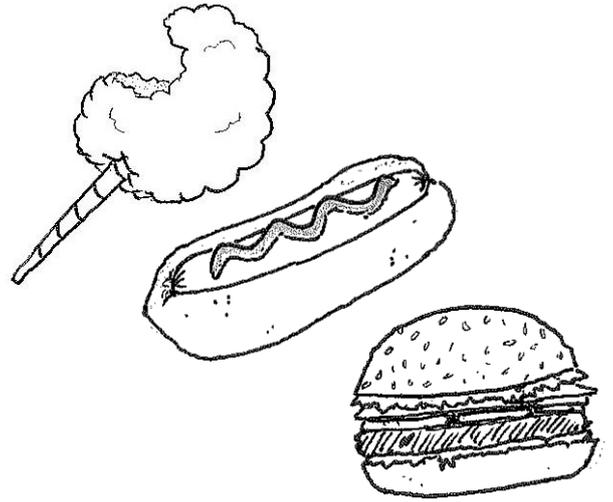
'Well,' said Michael, 'we spent £27 in total and mine was the dearest.'

'The product of the prices was 585,' added Charlie helpfully.

Sue still looked confused.

'My food was the cheapest item,' Anna told her kindly.

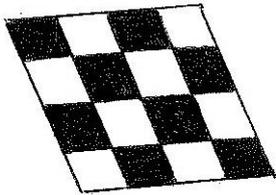
How much was each?



### 2. King's Procession

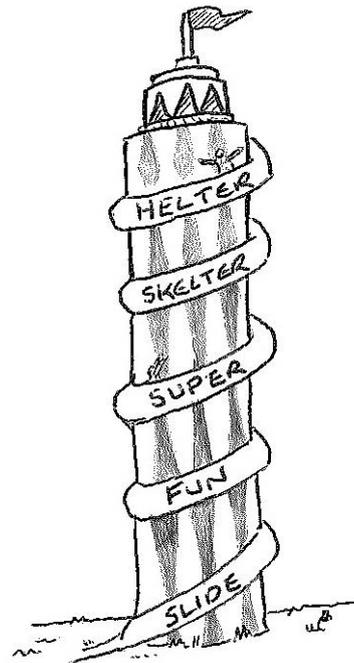
On a chessboard, a king is to be allowed to move one square at a time: horizontally to the right, vertically downward, or diagonally to the right and downward.

Imagine a reduced  $4 \times 4$  chessboard, with the king beginning in the top-left square. By how many routes can he reach the bottom-right square? By how many routes can a similar journey be made on a full  $8 \times 8$  chessboard?



### 3. Highly Scary Helter Skelter

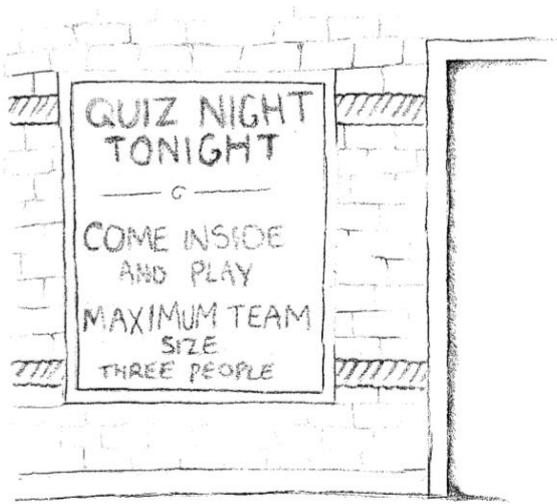
The highly scary helter skelter at the fair is a cylindrical tower of height 30 metres and circumference 8 metres. The slide is wound uniformly around the tower exactly 5 times. What is the total length of the slide?



#### 4. Domino Deception

Peter is playing dominoes with a standard 'double-six' set and begins by laying the double six. He then continues to lay dominoes, but in just one direction, until he has either exhausted his supply of 28 dominoes or cannot carry on. What will be the number at the end of the last domino? Why?

Suppose now that Peter plays in both directions from the double six. He discovers that he can use all the dominoes that he has available, but Ann announces that she hid one before he started. Can Peter always identify immediately (without checking the whole set!) which domino it will be?



#### 5. Quiz Night Quandry

A group of 11 friends heads to the pub on quiz night. The maximum size allowed for a quiz team is 3 and nobody wants to compete solo! In how many different groupings can the friends enter into the quiz?

#### 6. Archie and Phoebe

In the traditional game of hopscotch, children hop along a grid of squares drawn on the ground. At each position, there is either one square or two.

When one square is at a position, you land on one foot; when there are two squares, you land with one foot in each square.

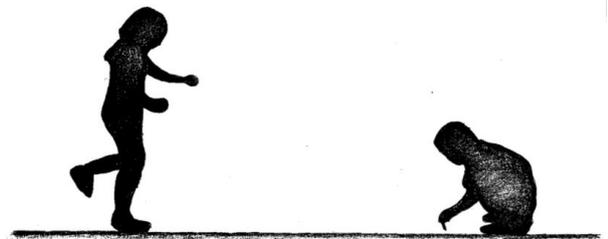
Archie and Phoebe want to play.

Draw all the different possible hopscotch grids they could make with four squares.

Now do it for grids of five squares.

A particular number of available squares will yield 34 possible grids. Thinking about any patterns between successive numbers of grids, predict this number of squares, explaining your reasoning.

(You do not need to draw out the 34 grids.)



## 7. Try Hard to Convert

Sarah, a keen rugby union player, has just scored a try in a match (a touchdown on the goal line) 5 metres to the left of the goal.

She gets up to attempt to score a conversion: this is a kick taken from the ground, where the ball must pass above a horizontal bar set between two vertical goal posts, which are 5.6m apart.

According to the rules, the conversion must be taken on a line extending backwards from the point of touchdown, perpendicular to the goal line.

Using a scale of  $1\text{cm} = 1\text{m}$ , draw a diagram to show from which point Sarah should take the conversion in order to maximise the angle subtended by the goal posts, thereby giving her the best possible view of goal. (This is the angle made by drawing lines from each goal post to meet at the point from where the kick is taken.)

## 8. Three Cups

'The hand is faster than the eye,' said James. 'Would you care to try?'

Alex is intrigued. 'What do I have to do?'

James replied, 'You pay £1 to play. I'm so confident that I'll place £2 underneath one of these three cups. I'll then swap around the cups at speed and you have to keep up. Identify the correct cup again and you win the £2, so you'd make £1 from me.'

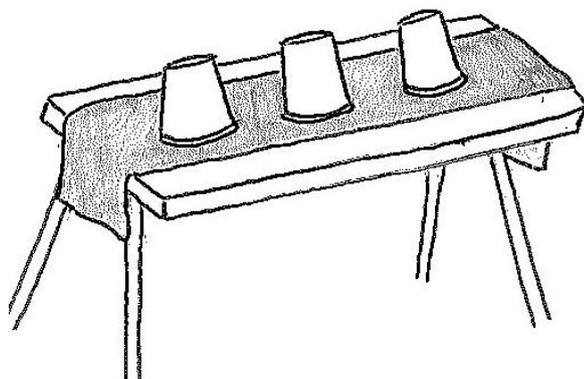
Alex agrees. Unfortunately, he has no idea where the £2 ends up and has to make a guess at random. James, who knows the location of the money, lifts up one of the two cups that Alex didn't select, showing Alex that there's nothing beneath it.

'Would you like to stick with your chosen cup or switch to the other remaining cup?'

Should Alex stick, switch or does it make no difference?

Assuming Alex guesses, but plays with the optimal strategy, explain whether the game is worth Alex's while to play.

James then offers Alex the chance to play for a £4.50 prize at a cost of £3. Should he accept?



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The competition is promoted by  
Mathematical Education on Merseyside (MEM)  
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