

MATHEMATICAL EDUCATION ON MERSEYSIDE

Sponsored by **MERCER**

Senior Challenge '16 For Year 10 or below

Illustrations by Theo Chaddock and Peter H Ackerley

Rules

- 1) Senior Challenge '16 should be attempted at home during February half term.
- 2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
- 3) Entries without any working out at all or written on this sheet **will not be marked**.
- 4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 5) You must write **your name and school in neat writing on every page.**

Either you or your maths teacher needs to return your entry by 4th March to this address:

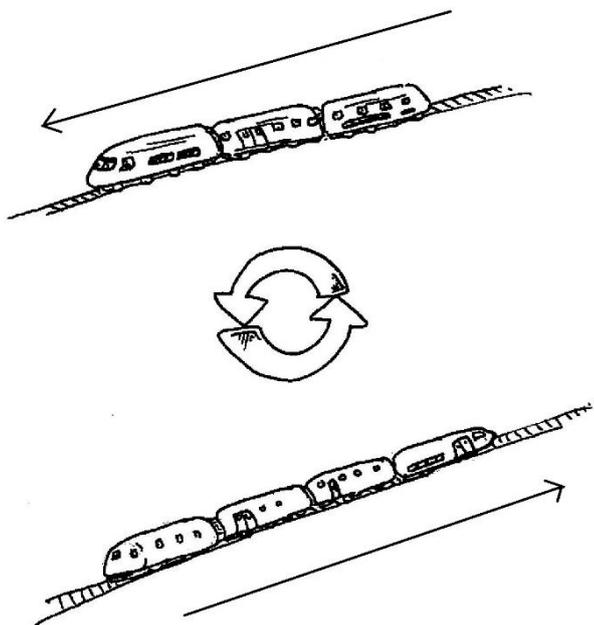
Senior Challenge '16 Entries,
Chris Marchant,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool,
L69 7ZL

A Prize-Giving Evening will be held at the University of Liverpool on 4th May.
We hope that you enjoy the questions.

1. Brum, Brum

Jo is setting out on a motorway drive to Birmingham in her rather elderly car, which now struggles when climbing! 10% of the distance Jo drives is uphill, where she can average only 56mph; 10% of the distance is downhill, where she can average 72mph; she averages 63mph on the flat. What is Jo's overall average speed for her whole journey?

Rachel drove the same journey in her flashy new car with cruise control. She realises that she drove for 10% of the journey time at 56mph, at 10% of the journey time at 72mph, and at 63mph for the remainder of the trip. Who travelled at the higher average speed for the whole journey?



2. Bettingland Line

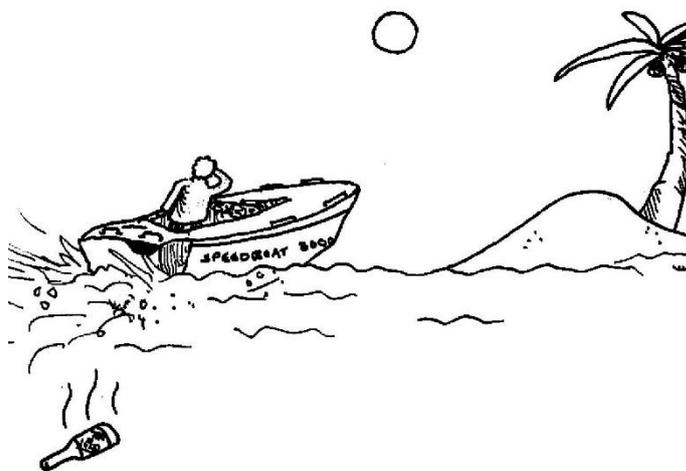
Trains run along the Bettingland Line between Betson and Winsome (via Hope) roughly once per hour in each direction. It is proposed to increase the frequency to be one train every 15 minutes in each direction.

A train from Betson to Winsome takes 50 minutes, whilst travelling from Winsome to Betson takes just 40, as it doesn't have to wait at Hope for a connection.

If the first train is to leave Betson at 0700 and the first train is to leave Winsome at 0715, how many trains are needed to run the service? How long will each train have to wait at each terminus?

3. Lost his Bottle

Will was travelling up a steadily flowing river ($4r$ was the river's flow in mph) in a boat fitted with a constant-speed outboard motor. The boat's speed in still water was $4v$ mph. He accidentally dropped a bottle overboard. Fifteen minutes later, he realised his loss, turned rapidly around, and started back downstream. He eventually caught up with the bottle after it had floated downstream for two miles. How fast was the river flowing?



4. Shunting Train

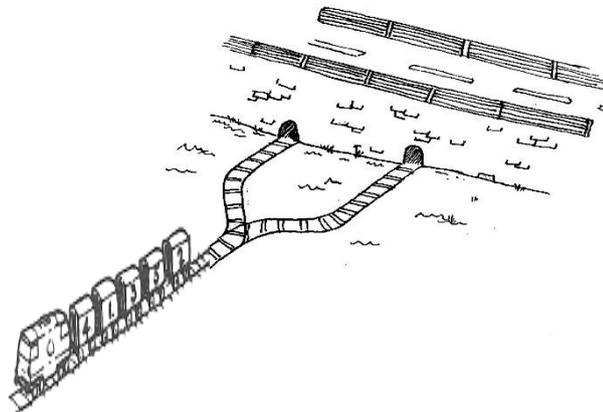
This diagram shows a train yard on a Heritage Railway which runs from West to East, with buffers at the end of each of the North and South branches.

A train standing on the West section consists of an engine and 5 trucks labelled 4, 1, 3, 5, 2 in that order. The driver needs to rearrange the trucks into the order 1, 2, 3, 4, 5.

The only 'moves' possible are either: rolling the train into a branch, removing the back truck or trucks, and moving the train back to the West section; or the engine collecting all of the trucks in the North branch and then all of the trucks in the South, before finally pulling the complete train back into the West section.

What is the shortest sequence of moves required to correctly order the trucks?

The next train arrives with trucks in the order 5, 4, 3, 2, 1. Show how to change this into the order 1, 2, 3, 4, 5 using as few moves as possible.



5. Lonnie and the Library

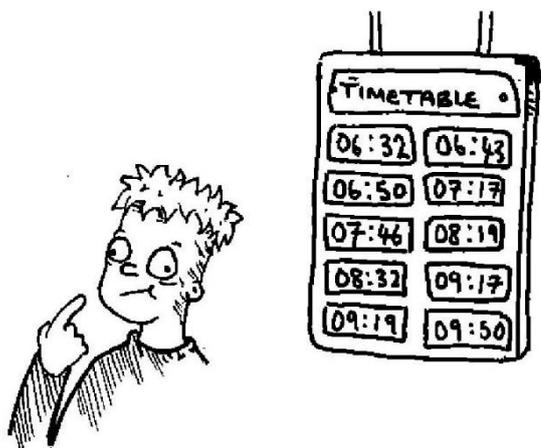
Lonnie is taking the train to the British Library in London. He tells Rosti the hour of his train's departure and he tells Ann at which minute it leaves. He also tells them both that the train leaves between 0600 and 1000.

They consult the timetable and find the following services between those times:

0632, 0643, 0650; 0717, 0746; 0819, 0832; 0917, 0919, 0950.

Rosti then says, 'I don't know when Lonnie's train leaves, but I'm sure that neither does Ann.' Ann replies, 'I didn't know his train, but now I do.' Rosti responds, 'Now I do as well!'

When is Lonnie's train and how do you know?



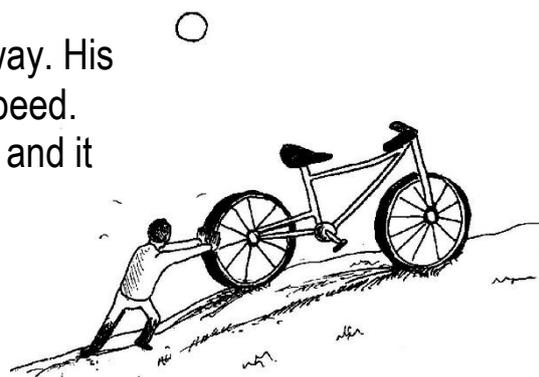
6. James' Journey

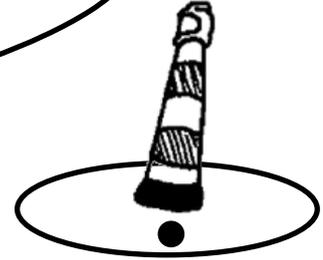
James has to cycle the four miles to university each day.

He starts out on Monday, but his bike gets a puncture partway (x miles along the route) and he has to push it the rest of the way. His walking speed, w , when pushing the bike is $\frac{1}{3}$ of his cycling speed.

On Tuesday, he only gets half as far before getting a puncture and it takes him $1\frac{1}{2}$ times as long to reach the university as it did on Monday.

On Wednesday, he doesn't get a puncture at all. What fraction of Monday's time, t , does he take to get there on Wednesday?





7. A Proper Charlie

Charlie has been out boating off the coast and is lost after a storm. He has a compass and is able to take bearings from his boat to three landmarks. His bearing to the church spire is 315° . His bearing to the windmill is 350° . His bearing to the lighthouse is 040° . He knows that his measurements are accurate to within 5° . He also knows that the church is due west of the windmill. Charlie radios these figures to the coastguard.

Trace the coastguard's map from this question paper. Use the information provided to draw on your map the search area that the coastguard should sweep to find Charlie.

8. Out of the Woods

Chris and his Scout Troop have been orienteering in the woods. They find themselves 200 metres from the nearest point, P, on a straight road which runs past the edge of the woods. Their minibus (M) is waiting to collect them 800 metres farther along the road from P. They can walk briskly along the road at 5km/h, but hike through the woods at only 3km/h. Let x be the distance in km from P to the point where they aim to meet the road. Plot a graph for x against the time t taken in minutes to reach M.

Use the graph to determine how far from M the Scouts should aim to meet the road in order to get there as fast as possible.

The competition is promoted by

Mathematical Education on Merseyside (MEM)
Registered Charity No 517028,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool,
L69 7ZL

MEM is proud to acknowledge generous financial support from the following during the past year:

The University of Liverpool
Liverpool Hope University
Liverpool John Moores University
Mercer Limited
The P H Holt Foundation
Liverpool Mathematical Society