MATHEMATICAL EDUCATION
ON MERSEYSIDE

Sponsored by MERCER

Senior Challenge '15

For Year 10 or below

Illustrations by Peter H Ackerley

Rules

1) Senior Challenge '15 should be attempted at home during February half term.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school in neat writing on every page.**

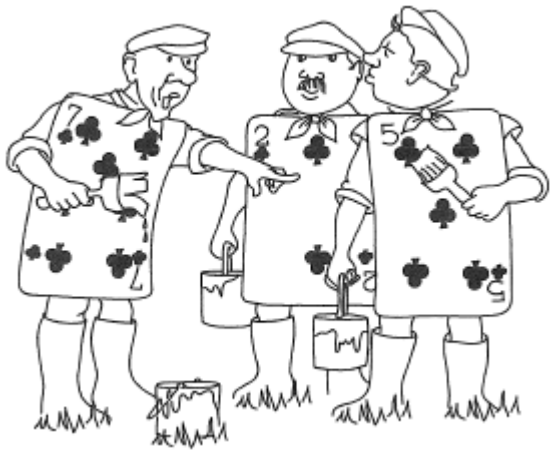
Either you or your maths teacher needs to return your entry by 6th March to this address:

Senior Challenge '15 Entries,
Chris Marchant,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool.
L69 7ZL.

A Prize Giving Evening will be held at the University of Liverpool on 29th April.
We hope that you enjoy the questions.

1. Roses are Red…

The Queen of Hearts hates white roses. Three of her gardeners were painting them red. They are living playing cards, specifically the Two of Clubs, Five of Clubs and Seven of Clubs.
The Two and the Five working together can paint 16 roses per hour.
The Five and the Seven can paint 18 roses between them in an hour.
The Two and the Seven can combine their labours to paint 10 roses per hour.
132 roses need to be painted, so how long would it take to accomplish this if all three playing cards cooperated?



2. A Pawn like Alice

In *Through the Looking-Glass*, Alice plays as a pawn on a giant square chessboard, which has 64 squares in total, arranged in equal numbers of ranks (rows) and files (columns). She has to reach the highest rank – the 8th rank – to be crowned a queen.
Imagine a second square chessboard (let's call this X) with an unknown number of squares.
A third square chessboard (Y) has 99 more squares than X, while a fourth square chessboard (Z) has 200 squares more than X.
For Alice to become a queen on chessboard X, which rank must she reach?

3. The Mad Hatter's Riddle

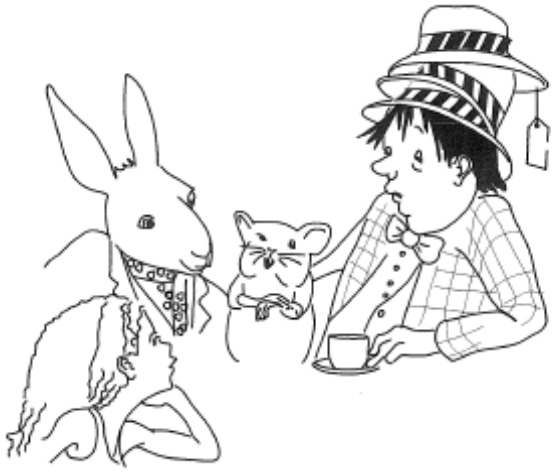
At the Mad Hatter's Tea Party, the other diners give poor Alice a series of increasingly contrived riddles.
The Hatter tells her that **14! = 871a82b12cd** and challenges her to find the four unidentified digits, in order to calculate the value of 14!.
Can you help her?
[Hint: 14! = 14 × 13 × 12 × 11 × 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1.]
Do not use a calculator or computer. You must carefully explain all of your logical reasoning.

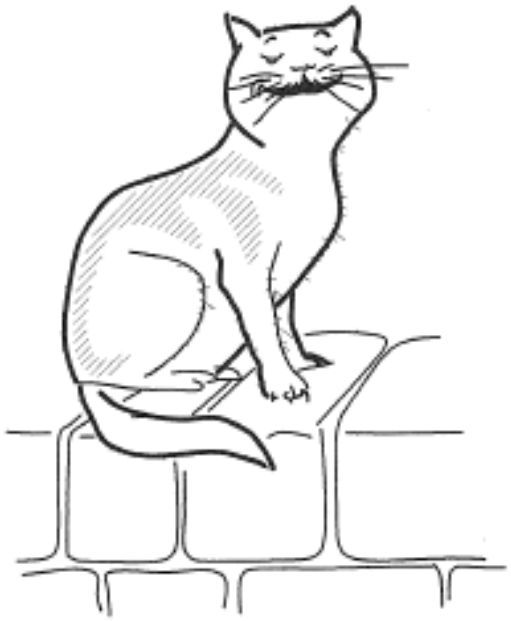
4. Chair Despair

The Mad Hatter's Tea Party, hosted by the Hatter, has three guests: Alice, the Dormouse and the March Hare.

He sets named places for all four diners, but is very angry to discover that everyone has sat down so that no one is sitting in the correct chair!

How many possible ways are there for this to happen? Explain why.





5. The Cat's Whiskers

The Cheshire Cat has made himself invisible to hide away from Alice and is sat on the perimeter wall of the Duchess's garden. The garden is rectangular in shape, with the perimeter walls running from North to South and from West to East.

He is located 5 metres from the North-East corner of the garden, 14 metres from the South-West corner, and 10 metres from one of the other two corners. The Northern wall is longer than the Western wall.

Use a scale of 1cm = 1m to draw diagrams, accurate to the nearest mm, showing the possible measurements of the Duchess's garden and marking the Cheshire Cat's location on each diagram.

How far is he from the fourth corner?

6. Jail Break!

To avoid their fate, 20 prisoners escape from the Queen of Hearts' jail into the population in her garden. In an effort to recapture them, the Queen has her soldiers seal off the garden immediately and then arrest 140 characters from the garden at random.

It is discovered that only 7 of those arrested were among her original prisoners! Estimate the total population of characters in the garden after the escape but before the new arrests. (Assume that the original prisoners did not get beyond the garden.)

7. Two Pairs of Squares

Lewis Carroll was actually the pen name of Charles Dodgson, whose day job was as a mathematician at the University of Oxford.

One entry in his diary mentioned his discovery that, when he doubled the sum of any two square numbers, the answer he obtained could always be written as the sum of two square numbers.

Show that this is true for 2^2 and 3^2.
Now try it for 5^2 and 7^2, then for 10^2 and 13^2.

What do you notice about the relationship between the two original square numbers in each case and the pair of square numbers that is equal to twice their sum?

Prove why this always works.

The solutions will be posted on www.maths.liv.ac.uk/~mem shortly after the prize giving evening to be held at the University of Liverpool on 29th April

The competition is promoted by
Mathematical Education on Merseyside (MEM)
Registered Charity No 517028
The Department of Mathematical Sciences,
University of Liverpool,
Liverpool,
L69 7ZL.

MEM is proud to acknowledge generous financial support from the following during the past year

Mercer Limited
The P H Holt Foundation
Liverpool Mathematical Society
The University of Liverpool
Liverpool Hope University
Liverpool John Moores University