MATHEMATICAL EDUCATION
ON MERSEYSIDE

Sponsored by MERCER

Senior Challenge '14

For Year 10 or below

Illustrations by Peter H Ackerley

Rules

1) Senior Challenge '14 should be attempted at home during February half term.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school in neat writing on every page.**

Either you or your maths teacher needs to return your entry by 7th March to this address:

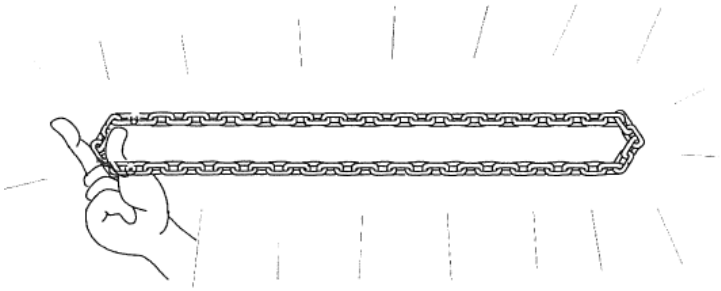
Senior Challenge '14 Entries,
Chris Marchant,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool.
L69 7ZL.

A Prize Giving Evening will be held at the University of Liverpool on 7th May.
We hope that you enjoy the questions.

1. Game for Olaf

Along a shoreline, running North-South, there are seaward-facing steep cliffs. There is a harbour in a narrow river mouth which forms a gap in the cliffs. Olaf, a Viking re-enactor, is on the open sea, somewhere to the East of the harbour. A thick fog descends, reducing visibility to just a few metres. Olaf has to sail towards the harbour using his compass. He knows that his compass is 5 degrees out from true, but can't remember in which direction. How should he proceed in order to safely get to the harbour?

2. Fenrir's Chain



Fenrir the wolf is bound by a magical chain. The chain is an endless piece made up of 30 links. It was originally forged as six pieces, each made up of 5 links. It costs 2 silver coins to cut a link and 4 to weld it together again. Find the cheapest way to create the endless chain.

3. Eye, Eye

Odin, Norse god of war, has only one eye according to their myths. An expedition of one-eyed Viking warriors sets out in a longboat from Norway, with some chickens and some cows. At this point, there are 70 pairs of legs aboard. They raid a settlement on the Scottish coast. 2 warriors are imprisoned by the Scots, but they manage to seize a cow and 5 chickens for the onward voyage. There are now 90 eyes on the longboat. By their arrival in Greenland, one more warrior has been buried at sea, 2 cows have been slaughtered for beef, but 2 eggs have hatched into chickens! 68 mouths dock at Greenland. How many Vikings, cows and chickens began the trip?

4. Viking Babies

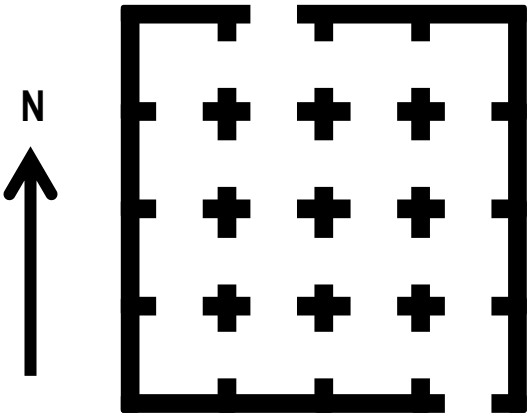
A Viking village is founded by a population of 100 couples. Each couple wishes to "continue the male line" but, as resources are scarce, they do not wish to overpopulate.

The agreed rule is that each couple has one baby per year, until the arrival of their first boy. For example, if, at some future date, a family has 5 children, then it must be 4 girls and a boy (the boy being the youngest and no more children planned), or 5 girls with another child planned.

Assume that children are equally likely to be born male or female.

Let p(n) denote the expected fraction of male children born in the nth year from the founding of the village.

Give a formula for p(n) for arbitrary positive n.



5. Escape the Monastery

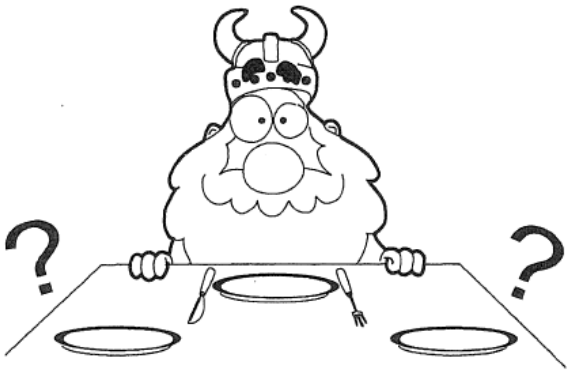
Mervin the Monk is running through the monastery to collect all of the treasure to take it to safety before the Viking raiders arrive (see the map on the left). He only has time to visit every cell once, and must start through the entrance door on the South side of the monastery and leave through the exit door on the North side. How many different routes are there where he visits all of the cells only once?

6. Land Ahoy!

Sigmund's long boat is sailing towards the shore. His wife, Freya, is standing on the beach (at sea level) waiting for her husband to arrive. As the sea is calm, Sigmund is standing in the crow's nest at the top of the ship's 15m mast. Given that the radius of the Earth is 6.4×10^6 m, and the long boat is sailing at a constant speed of 2m/s, for how long will Sigmund be able to see Freya before she can see the long boat's waterline? Assume they both are 1.5m tall and have perfect eyesight. Give your answer to the nearest minute.

7. Mead and Drink

Sven, a young Viking, has come of age and has outgrown his old helmet. A new one is made for him. You may assume that the basic helmet is a hemisphere, but with a cylindrical band attached underneath, which measures 60cm around the outside and is 5cm deep. What is the area of sheepskin needed to cover Sven's helmet? How many litres of mead could Sven drink if he were to fill his helmet to the brim (whilst holding it upside down)?



8. Eating Seating

Erik is holding a feast for himself and 9 other Vikings. As host, he will sit at the head of the long rectangular table; one guest will be seated at the foot of the table, with four on each side. Hagar doesn't want to sit at the foot. Olaf and Leif insist on being adjacent to one another and sitting on the side to Erik's left; Arvid, Balder & Colby are superstitious and refuse to be on the side to Erik's left. Delling, Frode and Gilby have no preference. How many arrangements are possible?

The solutions will be posted on www.maths.liv.ac.uk/~mem shortly after the prize giving evening to be held at the University of Liverpool on 7th May

The competition is promoted by
Mathematical Education on Merseyside (MEM)
Registered Charity No 517028
The Department of Mathematical Sciences,
University of Liverpool,
Liverpool,
L69 7ZL.

MEM is proud to acknowledge generous financial support from the following during the past year

Mercer Limited
The P H Holt Foundation
Liverpool Mathematical Society

The University of Liverpool
Liverpool Hope University
Liverpool John Moores University