## 7. Ways and Means

In a certain Solar System, the orbit of each planet has a known mean distance, in Astronomical Units (AU), away from the Sun. The mean distances of the orbits, working away from the Sun, form a geometric progression, where the common ratio is an integer (1, 2, 4, 8, ... is an example of a geometric progression with common ratio 2). The sum of the mean distances of all of the orbits of the planets is 1640, and the mean distances of the first and fourth orbits from the sun are 0.5 and 13.5 AU respectively. What is the largest of the mean distances of the orbits in the system?
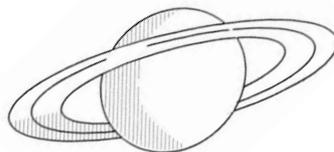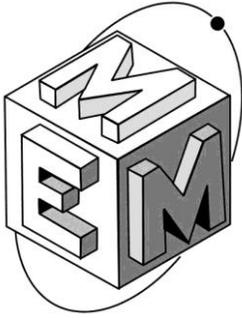
MATHEMATICAL EDUCATION ON MERSEYSIDE

Sponsored by **MERCER**

Senior Challenge '11
For Year 10 or below

Illustrations by Peter H Ackerley

Rules
1) It should be attempted at home during February half term.
2) Your entry must be your own work, though of course you may ask for help on how to start or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school in neat writing on every page.**

Either you or your maths teacher needs to return your entry by 11th March to this address:
Senior Challenge '11 Entries,
Chris Marchant,
Department of Mathematical Sciences,
University of Liverpool,
Peach Street,
Liverpool.
L69 7ZL.

All of the prizes will be awarded at an evening of mathematical recreation at the University of Liverpool on 11th May. Solutions will be posted on www.maths.liv.ac.uk/~mem shortly afterwards. We hope that you enjoy the questions.

1. Saturn the Bath

Saturn is so light in weight that it would float in a bath of water if one was big enough; its density is in fact 0.7kg/l. Given that Earth's volume is 1,084,000 km³, and Saturn's volume is 755 times larger than Earth's, how much does Saturn weigh?

2. Space Times

An old edition of 'THE TIMES ATLAS of the WORLD' is 16" × 10" and has 180 pages. The latest edition is 18" × 12½" and has 544 pages, including many details about Space! Clearly, it will need much more space on the bookshelf, but how much heavier would you expect it to be? Give reasons for your estimate.

3. Tour of Duty

At a base on Europa, a rectangular storage room, 32 yards × 48 yards, is divided by straight paths into six identical square areas. As a safety inspection, a robot has to traverse in turn each of the rectangular circuits possible in the room, but not starting the next circuit until the previous one is completed. The inspection starts by the Airlock at A, at one of the central crossings, and the finish is at the Bay door at B, at the middle of the shorter side farthest from A.

How many distinct rectangular circuits are there?

How many yards long is the inspection?
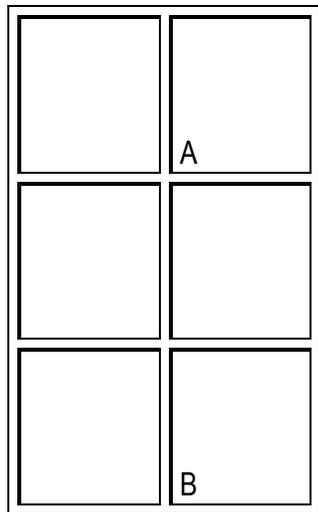
4. Work, Rest and Play
A Mars day lasts for 24.6 Earth hours. A Mars year lasts for 668.6 Earth days. How many Mars days are in a Mars year? Devise a leap year system for Mars to use up the extra part days.

5. Fuel Moon
A group of lunar buggies is stationed at a base at the Moon's North Pole. The tank of each buggy holds enough fuel to take it exactly half way around the Moon. Any amount of fuel can be transferred from the tank of one buggy to the tank of another when they are together. The only source of fuel is at the base. Assume that the buggies all have the same speed and fuel consumption throughout a journey, and that they all return safely to the base. What is the smallest number of buggies required to allow one buggy to complete a full circular trip around the Moon, passing over the South Pole?
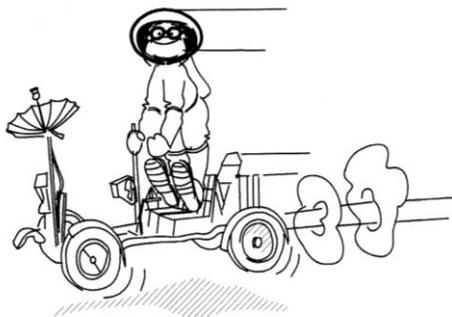
6. One Giant Leap
Luke Merseywalker is a high jump athlete with a Personal Best (PB) of 2.6 metres on Earth. He is selected to represent Earth at the 2112 Interplanetary Olympics on Titan. Based on his PB on Earth, what height would you expect him to achieve on Titan, given that the acceleration due to gravity there is 0.14g, where g its value on Earth? (The Games take place in a sealed Stadium Dome with an Earth-like atmosphere.)