

## 6. Knowing Me, Knowing You

Anna-Frid, Björn and Agnetha are stood in a line, one behind the other, looking in the same direction along the line. Anna-Frid can see both Björn and Agnetha; Björn can only see Agnetha, who cannot see either of the other two. They are blindfolded and a hat is placed on each of their heads. They are told that the hats are selected at random from a box containing two blue hats and three white hats. The blindfolds are removed, and they are asked to state, without turning around or conferring, what colour hat they themselves are wearing. After a pause, during which no-one has spoken, Agnetha announces that she knows the colour of her hat. What is the colour of her hat and how does she know this?



## 7. Merry Go Round

A circular disc can be divided into twelve 3-sided pieces, all of the same shape & size. Only 6 of them meet at the centre of the disc. Figure out how this is done.

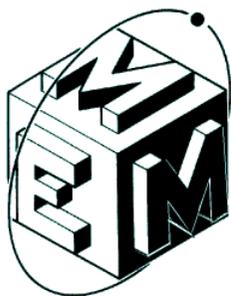
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The competition is promoted by  
Mathematical Education on Merseyside (MEM)  
Registered Charity No 517028  
The Department of Mathematical Sciences,  
University of Liverpool,  
Liverpool,  
L69 7ZL.

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# MATHEMATICAL EDUCATION ON MERSEYSIDE

Sponsored by **MERCER**

# Senior Challenge '09 For Year 10 or below

Illustrations by Peter H Ackerley

## Rules

- 1) It should be attempted at home during February half term.
- 2) Your entry must be your own work, though of course you may ask for help on how to start or for the meanings of unfamiliar words.
- 3) Entries without any working out at all or written on this sheet will not be marked.
- 4) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 5) You must write **your name and school in neat writing on every page.**

Either you or your maths teacher needs to return your entry by **6<sup>th</sup> March** to this address:

Senior Challenge '09 Entries,  
Chris Marchant,  
Department of Mathematical Sciences,  
University of Liverpool,  
Peach Street,  
Liverpool.  
L69 7ZL.

All of the prizes will be awarded at an evening of mathematical recreation at the University of Liverpool on **4<sup>th</sup> June**. Solutions will be posted on [www.maths.liv.ac.uk/~mem](http://www.maths.liv.ac.uk/~mem) shortly afterwards. We hope that you enjoy the questions.

## 1. Money, Money, Money

Eight coins are arranged in a row on a table. In a game, they need to be moved to create four stacks, each of two coins. The rule is that each time a coin is moved, it must jump over exactly two coins and be put down on to a single coin. What is the minimum number of moves required? Show how you achieve this.



## 2. The Winner Takes It All

In a knock-out chess tournament, the first round, consisting of a number of matches and a number of byes, has been drawn up so that there will be 32 competitors in the second round. Four late entries arrive, and are added into the draw. This time, the number of matches is equal to the number of byes that there were in the first draw. What is the final number of entries?

## 3. Ring Ring

Benny is asked by a market researcher for his phone number; feeling that this is a little intrusive, he replies with the following: "I have a Liverpool phone number, which can be written in three parts as 0151-abc-defg. One of the two parts you have to find is a three-digit number, and the other a four-digit number. Each of these parts is both of the form  $n! + 1$  for some  $n$  and of the form  $m^2$  for some  $m$ , with  $n$  and  $m$  both positive integers, which may be different for each part." What is Benny's phone number?





#### 4. Summer Night City

One hot summer night, in a city tower block with 50 floors, a bored electrician at the fuse board turns the power off to all of the floors. He then switches the power back on to every second floor. Then he goes through the switches in the same order and changes the position of every third switch; i.e. if it is on, he turns it off, and vice versa. He repeats this with every fourth switch, every fifth and so on. When he has finished, how many floors will be without power and, if the floors are numbered in the American style from 1 to 50, which floors would remain without power?

#### 5. Take a Chance on Me

Fernando flips a fair coin repeatedly. After each flip, he records the uppermost face shown by the coin, head or tail, as H or T, and thus creates a random sequence of H's and T's. Fernando stops flipping and terminates his sequence when either of the two sub-sequences HHH or THH first occurs. Which of these two sub-sequences would you anticipate to be the one more likely to terminate his sequence, and why?

