

SENIOR CHALLENGE '08 - Solutions

1. STARTER

The numbers are 15 and 20.

2. HOTEL BABBLE-ON

1	2	3	4	5	6
B	R	S	M	L	V

Victoria is in Room 6.

3. ROW, ROW YOUR BOAT

1. ABC cross -- 4 minutes
2. AB go back – 6 minutes
3. ADE cross -- 4 minutes
4. A goes back -- 11 minutes
5. ABF cross -- 4 minutes

That's 29 minutes in all. Plenty time to catch the train.

There is another solution, involving two more crossings, that takes five minutes more, also OK.

4. CEMENTED

$r = 150$ mm

5. DOES IT DEPEND?

Let L be the side length of the triangle and A, B, C the perpendicular distances from P to the sides. Then the area of the triangle, as the sum of the areas of three triangles, is one half of $L(A + B + C)$. So $A + B + C$ is independent of the position of P , which is what had to be proved.

6. DIGITAL 2008

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7. INTRODUCTION TO GROUPS

The powers of 2 modulo 5, starting with the 0^{th} , are 1, 2, 4, 3, 1,.....

So 2 generates the group.

The powers of 2 modulo 11 are 1, 2, 4, 8, 5, 10, 9, 7, 3, 6, 1,.....

So 2 generates the group.

The powers of 2 modulo 41 are

1, 2, 4, 8, 16, 32, 23, 5, 10, 20, 40, 39, 37, 33, 25, 9, 18, 36, 31, 21, 1,....

So 2 is **not** a generator of the group.

The powers of 3 modulo 41 are

1, 3, 9, 27, 40, 38, 32, 14, 1,.....

So 3 is **not** a generator of the group.

Note that in each case $40 = -1$ occurs halfway along the list.

The first 11 powers of 5 modulo 41 are

1, 5, 25, 2, 10, 9, 4, 20, 18, 8, 40,....

So 5 is **not** a generator of the group.

The first 21 powers of 7 modulo 41 are

1, 7, 8, 15, 23, 38, 20, 17, 37, 13, 9, 22, 31, 12, 2, 14, 16, 30, 5, 35, 40, ...

So 7 **is** a generator of the group.

(Actually 6 also is a generator, with $7 \times 6 = 1$ modulo 41.)

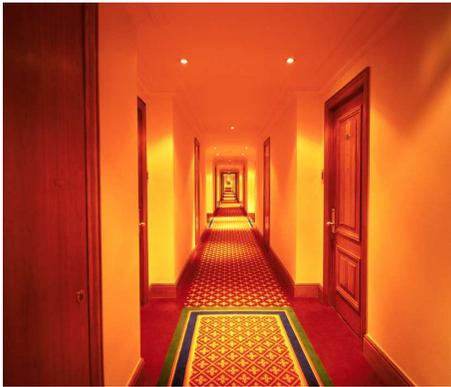
It is a theorem, requiring careful proof, that a generator always exists for each prime number p . The classic text, for many a first introduction to the joys of Number Theory, is 'The Higher Arithmetic', by J.H. Davenport, CUP.

More detailed solutions to some of these questions may be posted later.

1. STARTER

The number 625 is the sum of the of two 2-digit numbers. The difference between the numbers is one third of one these numbers and one quarter of the What are these two numbers?

625 squares
of
other.



2. HOTEL BABBLE-ON

Six friends are in six hotel rooms, numbered from 1 to 6. The rooms are all on the same side of the corridor.

Lizzy is next to Victoria.

Mike's room number is double Rick's.

Brian is next to only one of his friends.

Lizzy's room is a prime number.

Brian's room number is a third of Sue's.

What was the number of Victoria's room?

3. ROW, ROW, ROW YOUR BOAT

The six friends leave the hotel to catch the last train home, which leaves in 40 minutes.

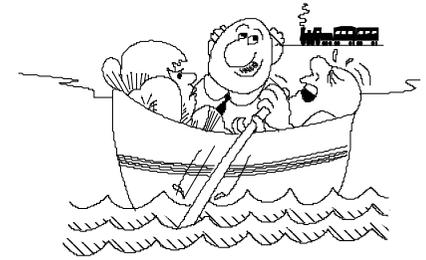
However before reaching the station they must cross a deep river. To do this they

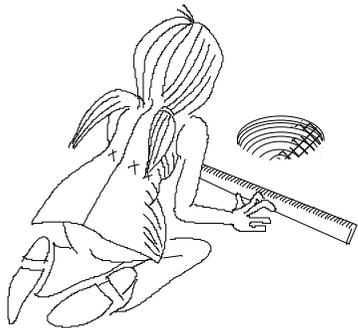
must use a rowing boat that holds only

three people, but has a paddle as well as

two oars! It takes one person 11 minutes to row across the river in either direction, two people 6 minutes and three people 4 minutes.

From the far bank, it will take 6 minutes to get to the station. None of the friends can swim, and they cannot leave anyone behind. Can they make it to the train, and if so, how?





4. CEMENTED

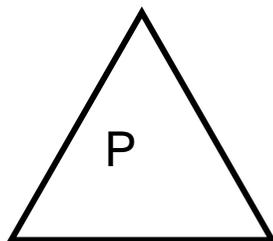
As Sjaan walked down the road, she saw a spherical ball partially stuck in cement. She removed the ball to find that the hole in the cement is 60mm deep and 240mm wide. What is the radius of the ball?

There is a similar group G_p where the number 5 is replaced by any other prime number, p .

Find a generator for G_{11} and one for G_{41} .

5. DOES IT DEPEND?

Let P be a point somewhere inside an equilateral triangle. Does the total of the perpendicular distances from P to each of the three sides of the triangle depend on the position of P ?



6. DIGITAL 2008

Let w be a 2008-digit number divisible by 9.

Let x be the sum of the digits of w .

Let y be the sum of the digits of x , and z the sum of the digits of y .

What is the sum of the digits of z ?

7. INTRODUCTION TO GROUPS

The **group of integers modulo 5**, G_5 , consists of the numbers 1 to 4, with the rule that the product of any two of them is the remainder when their ordinary product is divided by 5. Verify that each of these four numbers is a power of 2.

We say that 2 is a **generator** of the group.