Senior Challenge 07 Answers

1. A BARGAIN WITH THE DEVIL

Let x be the initial amount of money:

After 1 bridge he has: $2x-24$

After 2 bridges he has: $2(2x-24)-24=4x-72$

After 3 bridges he has: $2(4x-72)-24=8x-168=0$

$\Rightarrow 8x=168 \qquad \Rightarrow x=21$ i.e. He started with 21p.

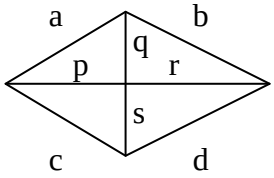
2. TAKE A BET?

The finishing order is Black, Crystal, Azalea.

The other 5 permutations lead to contradictions.

3. A STICKY PROBLEM

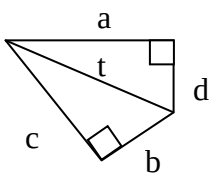
Let a, b, c and d be the 4 sticks. Then:



$a^2=p^2+q^2 \qquad b^2=r^2+q^2$

$c^2=p^2+s^2 \qquad d^2=s^2+r^2$

The sticks can now be rearranged to give



$a^2=p^2+q^2 \qquad b^2=r^2+q^2$

$c^2=p^2+s^2 \qquad d^2=s^2+r^2$

$t^2=a^2+d^2=p^2+q^2+r^2+s^2=p^2+s^2+r^2+q^2=b^2+c^2$

Thus the two hypotenuses are the same and you have 2 right angled triangles combining to form a quadrilateral. This is an exercise in Pythagoras' Theorem and its converse i.e. that if Pythagoras' Theorem works, then you have a right angled triangle. The above diagrams are not to scale.

4. MEANS TO AN END

Olga is calculating exact fractions, not decimal approximations. In fact the numbers she computes each converge to the square root of two. The product is always exactly equal to 2.

$$\frac{A+B}{2}\times\frac{2AB}{A+B}=AB$$

This is equal to 2 on the 1st January, and remains so throughout the month.

5. SOLOMON'S BOWL

Internal diameter = $30/\pi \approx 9.55$ cubits, so internal radius = $15/\pi$

External Circumference = $10\pi \approx 31.4$ cubits, so external radius = 5

Volume of larger hemisphere = $\frac{1}{2}\times\frac{4}{3}\times\pi\times5^3=\frac{4}{6}\times\pi\times125=\frac{500\pi}{6}\approx261.75$ cubits³

Volume of smaller hemisphere = $\frac{1}{2}\times\frac{4}{3}\times\pi\times\left(\frac{15}{\pi}\right)^3=\frac{4}{6}\pi\times\frac{3375}{\pi^3}=\frac{2250}{\pi^2}\approx227.97$ cubits³

Volume of Bowl = $\frac{500\pi}{6}-\frac{2250}{\pi^2}=\frac{500\pi^3-13500}{6\pi^2}\approx33.78$ cubits³ (to 2dp)

6. 2007 BOOK

The only correct statement is:

"This book contains exactly 2006 false statements"

Any 2 or more statements being correct lead to contradictions.

7. INVITATION CARDS

There are 45 (10×9÷2) pairs and 50 cards. There is therefore a minimum of 5 couples.

As part of the Senior Challenge evening on 5th June, we will present a full set of solutions.