

# MATHEMATICAL EDUCATION ON MERSEYSIDE

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Human Resource Consulting

## Senior Challenge '07 For Year 10 or below

Illustrations by Peter H Ackerley

### Rules

- 1) The competition is open to all pupils in and around Merseyside who are in Year 10 or below.
- 2) It should be tackled at home, during February half term.
- 3) Your entry must be your own individual effort, though of course you may refer to books, the internet or your teachers on how to start or for the meanings of unfamiliar words.
- 4) Marks will be awarded to clearly explained solutions only.
- 5) We hope that you enjoy the questions.
- 6) It is possible to win a prize even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 7) Hand your neatly written or printed entry **with your name and school on every page**, to your maths teacher as soon as possible after the February half term so they can send them to us by 9<sup>th</sup> March.
- 8) All of the prizes will be awarded at an evening of mathematical recreation at The University of Liverpool on 5<sup>th</sup> June.
- 9) Solutions will be posted on [www.maths.liv.ac.uk/~mem](http://www.maths.liv.ac.uk/~mem) by the end of March.

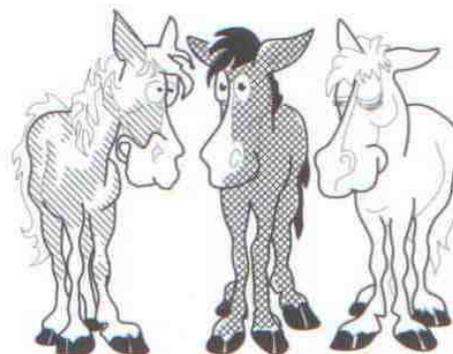
### 1. A BARGAIN WITH THE DEVIL

Once upon a time the Devil suggested to a lazy man how to earn some easy money: "At each bridge on your journey I'll double your money and after crossing the bridge you shall only give me 24 pence back." The lazy man agreed, but had lost all his money after the third crossing. How much money did he have before agreeing the deal?



### 2. TAKE A BET?

Three horses Azalea, Black and Crystal are taking part in a race. Before the race Xavier claimed that Azalea would finish first. Yuri answered: "Black couldn't come first anyway". "But Crystal wouldn't be the last" - said Zack. After the race it turned out that only one of them was correct. In what order did the horses actually finish?



### 3. A STICKY PROBLEM

Amelia has four sticks of different lengths. She knows that they can be arranged to form the sides of a convex quadrilateral, whose diagonals are mutually orthogonal (at right angles to each other). Show her how to form from them a quadrilateral, two of whose angles are right angles.





#### 4. MEANS TO AN END

Recently Olga decided to improve her arithmetical skills. So, on New Year's Day she wrote the two numbers 1 and 2 in her new diary. Every day she then computed two new numbers, the *arithmetic mean* and the *harmonic mean* of the two numbers written on the previous day.

Find the product of the two numbers that she wrote in her diary on 31<sup>st</sup> January.

[The *arithmetic mean* of two numbers  $A$  and  $B$  is the simple average  $\frac{A+B}{2}$ , while their *harmonic mean* is the reciprocal of the arithmetic mean of the reciprocals  $1/A$  and  $1/B$  of  $A$  and  $B$ , which you may verify is equal to  $\frac{2AB}{A+B}$ ]

equal to  $\frac{4}{3}\pi r^3$ ]

#### 5. SOLOMON'S BOWL

According to the Bible, in Solomon's Temple there was a hemispherical gold bowl, which measured 30 cubits around the circumference and 10 cubits across.

This appears to be a problem if  $\pi = 3.1415\dots$  However, the diameter measured is in fact the *external* diameter of the bowl, whilst the circumference is around the *inside* of the rim of the bowl.

First, calculate the internal diameter and the external circumference. Then calculate the volume of gold needed to make the bowl.

[The volume of a sphere of radius  $r$  is



[The volume of a sphere of radius  $r$  is

## 6. 2007 BOOK

A book entitled "2007 BOOK" contains  
2007 statements:

"This book contains exactly 1 false statement."

"This book contains exactly 2 false statements."

... ..

"This book contains exactly 2007  
false statements."

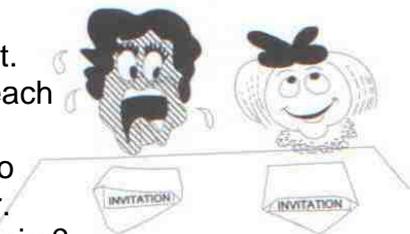
Which statements in this book are true?

## 7. INVITATION CARDS

There are 10 pupils in the top maths set.  
Each pupil sends an invitation card to each  
of five other pupils in the set.

Prove that there must be two pupils who  
have sent invitation cards to each other.

What is the minimum number of such pairs?



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