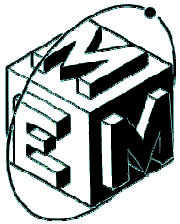
"The greatest mistake you can make in life is to be continually fearing you will make one." - **Elbert Hubbard**



MATHEMATICAL EDUCATION ON MERSEYSIDE

Senior Challenge 2006

For Ages 15½ and Under

Sponsored by MERCER Human Resource Consulting

Illustrated by **Peter H Ackerley**

Notes

1) The competition is open to all pupils at schools in and around Merseyside who are under the age of fifteen and a half (born on or after 1st September 1990). It is to be tackled at home, during the February half-term.

2) Your entry must be your own unaided effort, though of course you may refer to books, etc., for ideas on how to start, and may ask the meanings of unfamiliar words. *More marks will be given if you explain clearly how you get your answers.*

3) We hope that you enjoy the questions. It is possible to win a prize even though you may not have attempted all of the questions, so do let us have your entry even if it is not quite finished!

4) Hand your neatly written entry, with your name on every page, to your class teacher as soon as possible after the February half-term.

5) Prizes for overall winners and many consolation prizes will be presented at an Evening of Mathematical Recreation at the University of Liverpool in May. Certificates will be awarded to all who do well.

6) Solutions will be posted on www.maths.liv.ac.uk/~mem early in March.

The competition is promoted by Mathematical Education on Merseyside (MEM)
Registered Charity No 517028
The Department of Mathematical Sciences, The University of Liverpool, Liverpool, L69 7ZL

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Alliance & Leicester ● Mercer Human Resource Consulting ● The Holt Charitable Trust
The University of Liverpool ● Liverpool Hope University College

1. Prime starter

Prove that there is one and only one way to express any odd prime number as the difference of two squares. How are these squares then related?

2. Uncommon children

This concerns a group of four children, any two of whom have something in common: first name, surname, or school year. Can we then be sure of finding three children with the same thing in common?

3. Trapped square

Draw a triangle with sides 3, 4 and 5, and inside this draw the largest square that will fit. What is the area of the triangle? What is the area of the square?

4. Bacterial warfare

A biologist keeps dangerous bacteria of six different kinds in a long glass tube separated into six chambers by membranes. She has noticed that, if she removes a membrane, bacteria start fighting their neighbours. Those which are in a minority always die, together with an equal number of the enemy. Equal colonies eat each other completely! Originally there were a million bacteria of the first kind, two million of the second, and so on, up to six million of the sixth kind. Was it possible for her to open the membranes in turn in some order so that no bacteria survived?

5. Integral Triangle

(i) Find a triangle, not right-angled, each of whose sides is an integer, and whose area also is an integer.

(ii) What do you notice about the title of this problem?

6. Challenging hexagon

Take the triangle you made in Problem 5, erect a square on each of its sides (outside the triangle), then draw three lines connecting corners of the squares to form a hexagon that is made up of three squares and four triangles. The area of this hexagon will again be an integer. Why? Find this area.

For this last one you need to know a little about the sine of an angle, and the formula

$$\text{Area} = {}^1\!/_2 \, bc \sin A$$

for a triangle with sides a, b and c, the angles opposite these sides being A, B and C.

This is an opportunity to explore ahead of the syllabus!

Enjoy the experience.