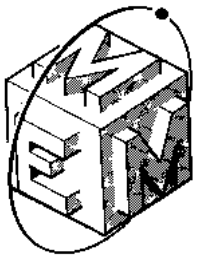


CHALLENGE '05

Sponsored by Alliance & Leicester Commercial Bank

1. 1105

The sum of the squares of two consecutive numbers is 1105. What are the two numbers?

Solution: 23 and 24.

2. COACHES

A coach operator, not having room in his garage for eight of his coaches, increased the size of the garage by 50%, and then had room for eight more coaches than the number he owned.

How many coaches did he own?

Solution: 40 coaches.

3. NARCISSISTIC NUMBERS,

Calculate

$$1^3 + 5^3 + 3^3.$$

What is special about this number?
Find at least two other similar 3-digit numbers. (One at least of them involves the digit 7.)

Solution: The given number is 153, equal to the sum of the cubes of its digits. Other numbers sharing this property are 370, 371 and 407.

4. MAY AND SEPTEMBER

An old man married a young woman. Their combined ages amounted to 100. The man's age multiplied by 4 and divided by 9 gave the woman's age.
What were their respective ages?

Solution: 69 years and 12 weeks and 30 years and 40 weeks.

5. PALINDROMIC EQUALITIES

Did you know that certain pairs of two-digit numbers have the same product when both numbers are reversed? A *trivial* pair is one where the second number is the reverse of the first. An example is

$$12 \times 21 = 12 \times 21.$$

Non-trivial examples are

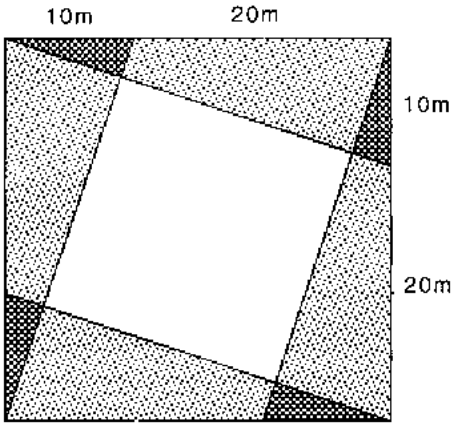
$$12 \times 42 = 24 \times 21 \qquad 12 \times 63 = 36 \times 21 \qquad 13 \times 62 = 26 \times 31$$

$$23 \times 96 = 69 \times 32 \qquad 24 \times 63 = 36 \times 42 \qquad 69 \times 64 = 46 \times 96.$$

There are eight other such non-trivial pairs. Try to find them all.

Solution:

$$12 \times 84 = 84 \times 21 \qquad 13 \times 93 = 39 \times 31 \qquad 14 \times 82 = 28 \times 41 \qquad 23 \times 64 = 46 \times 32$$
$$24 \times 84 = 48 \times 42 \qquad 26 \times 93 = 39 \times 62 \qquad 34 \times 86 = 68 \times 43 \qquad 36 \times 84 = 48 \times 63.$$



6. CARPET-BEDDING

According to the dictionary *carpet-bedding* is a 'system of horticulture in which plants are arranged in a geometrical mosaic'.
The garden whose plan is given to you is a 30 metre by 30 metre square. The paved square in the centre is surrounded by a number of triangular flower beds, four of which are shaded black, the grey areas being filled with other triangular flower beds the same size and shape as these four.
What fraction of the whole is the square in the centre?

Solution: The small dark triangles each have area $15m^2$, the larger similar triangles that are the unions of dark triangles and grey trapezia being linearly three times the size and areally nine times the size, the trapezia each having area $120m^2$. The flower beds have total area $540m^2$ and the paved area $360m^2$, which is $\frac{2}{5}$ of the whole.

7. POCKET MONEY

A father divided a certain number of pounds among his four children. To the first he gave a part, to the second one third of what was then left after the first's share, to the third he gave five-eighths of what was then left, and to the fourth the balance, which equalled two-fifths of the first child's share.
Each child received a whole number of pounds.
No child received as much as £20.
How much money did the father distribute, and how much did each child receive?

Solution: The father distributed £39, the children receiving respectively £15, £8, £10 and £6.