

**MERCER**

Human Resource Consulting

## *SENIOR CHALLENGE '03*

1. This competition is open to all pupils at Schools in and around Merseyside who are under the age of fifteen and a half (born on or after 1 September, 1987). It is to be tackled at home, during half-term.
2. Your entry must be your own unaided effort, though of course you may refer to books, etc., for ideas on how to start, and may ask the meanings of unfamiliar words. *More marks will be given if you explain clearly how you get your answers.*
3. We hope that you enjoy the questions. It is possible to win a prize even though you may not have attempted all the questions, so do let us have your entry even if it is not quite finished!
4. Hand your neatly written entry, with your name on every page, to your class teacher as soon as possible after half-term.
5. Prizes for overall winners and many consolation prizes will be presented at an Evening of Mathematical Recreation at the University of Liverpool in May. Certificates will be awarded to all who do well.
6. Solutions will be posted on [www.liv.ac.uk/maths/MEM/](http://www.liv.ac.uk/maths/MEM/) early in March.

### **1. CUISENAIRE**

Suppose you have rods with lengths 1, 2, 4, 8. Then, apart from these lengths you can make quite a few others, for example  $3 = 2 + 1$ ,  $5 = 4 + 1$ ,  $7 = 4 + 2 + 1$ . What in fact is the shortest whole number length you *cannot* make with these four rods?

### **2. UNEXPECTED GUEST**

Giovanni and Leporello invite four young ladies to a quiet evening of pizza and Karaoke. They cut the circular pizza into six equal pieces and are about to start eating when an unexpected guest arrives. Leporello cuts equal amounts off the six pieces to give to the guest so that all seven people have the same amount of pizza. What fraction of each piece did he cut off?



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The Department of Mathematical Sciences, The University of Liverpool, Liverpool, L69 7ZL.

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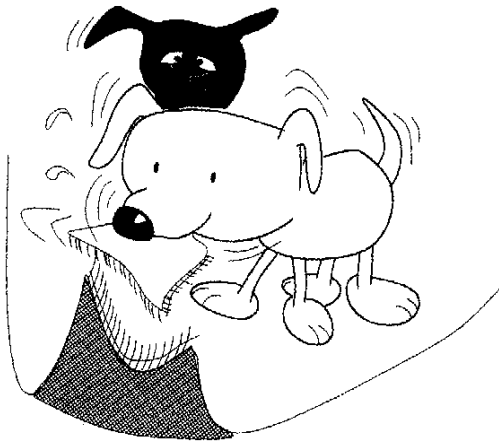
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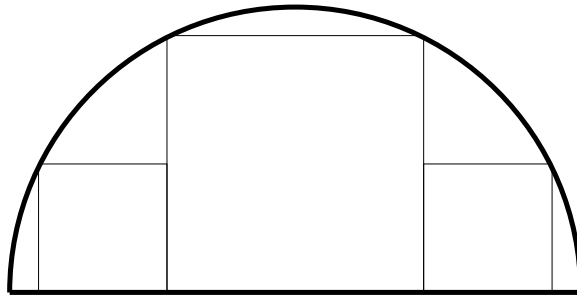


### 3. GOOD DOGS

‘Be good dogs,’ said Mr Sumhope as he left Fido and Trusty to guard his house while he was out. When they were alone, the two dogs started to tear the living room carpet into pieces. When Fido chose a piece he tore it into four parts, and when Trusty chose a piece she tore it into seven parts. Being good dogs, they never chose the same piece at the same time. When Mr Sumhope returned he found 2003 pieces of carpet. Were there any missing?

### 4. GARDENERS’ QUESTION TIME

Mrs Pythagoras is making a semicircular patio, as in the figure, with three squares exactly fitting as shown. These squares are to be covered with paving stones and the rest of the semicircle, outside the squares, is for planting flowers. Given that the diameter of the semicircle is 10m, how much area is left for planting?

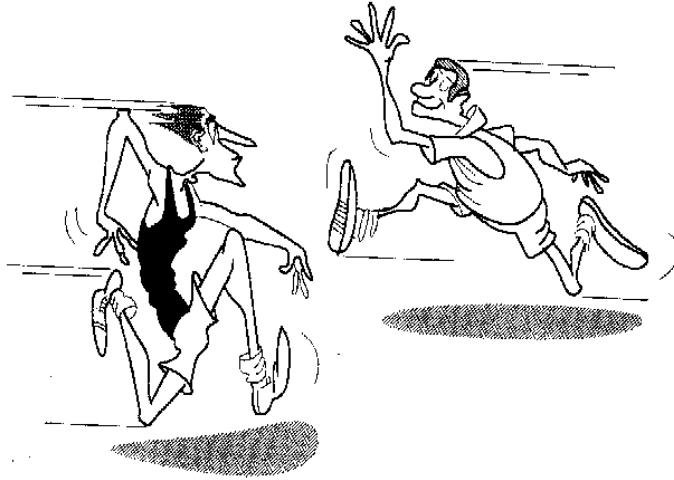


### 5. DOING THE SPLITS

Amanda and her brother Gareth were playing with six-figure numbers, splitting them in the middle to make two three-figure numbers. Amanda found one six-figure number which was exactly 7 times what she got by multiplying together her two three-figure numbers. Find this six-figure number if you can. (Hint: 1001 is divisible by 7.)

Gareth looked for a six-figure number which actually equalled what he got by multiplying together his two three-figure numbers. Do you think he succeeded?





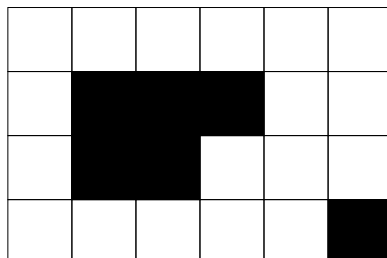
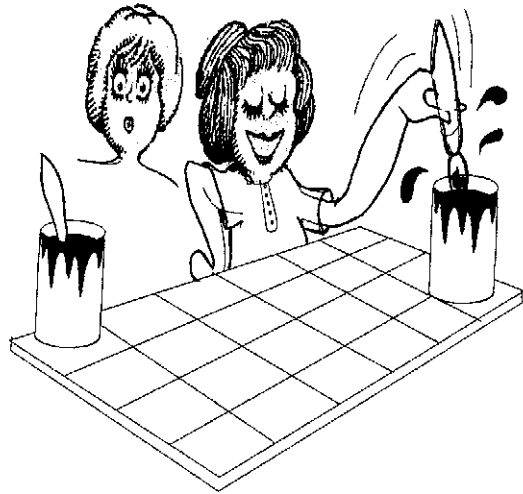
## 6. RUNNING MATES

Two runners, Chris and Alex, each running at his own constant speed, leave their houses at the same time, each running towards the other's house. They meet 600m from Chris's house but continue running, turning round as soon they they reach each other's house and meeting again 400m from Alex's house. How far apart are the houses?

## 7. SQUARE BASHING

Two clever painters Abe and Beth are playing the following game. They start with a table having marked on it a  $6 \times 4$  grid of squares. They take turns and at each turn must paint a square of any size with the edges along the grid lines and not overlapping the previously painted area.

For example, the figure shows a possible position after Abe painted a  $2 \times 2$  square and Beth painted a  $1 \times 1$  square next to it and Abe then painted a  $1 \times 1$  square in the corner.



The person to paint the last piece of the table wins. Starting from scratch, who should win?

What about other sizes of grid such as  $5 \times 7$  or  $10 \times 20$ ? (You may assume that both dimensions are even or both are odd.)